HONORS GEOMETRY SUMMER HOMEWORK

NAME ______________________________

ALGEBRA TEACHER ______________________________

SOLVING LINEAR EQUATIONS (SINGLE STEP)

The equations $\frac{1}{2}x = 4$ and $3t - 1 = 4t$ are examples of **linear** equations. When the variable in a single-variable equation is replaced by a number and the resulting statement is true, the number is a **solution** of the equation. You can *solve an equation* by writing an *equivalent* equation that has the variable alone on one side. One way to do this is to add or subtract the same number *from each side* of the equation.

EXAMPLES Solve the equation.

a. $x + 6 = -2$

b. $y - 7 = 3$

SOLUTION

a.

$x + 6 = -2$

$x + 6 - 6 = -2 - 6$ Subtract 6 from each side.

$x = -8$ Simplify.

b.

$y - 7 = 3$

$y - 7 + 7 = 3 + 7$ Add 7 to each side.

$y = 10$ Simplify.

Another way to solve a linear equation is to multiply or divide each side by the same nonzero number. Notice the use of reciprocals in the example below.

EXAMPLE Solve the equation $8 = \frac{4}{3}a$.

SOLUTION

$8 = \frac{4}{3}a$

$\frac{3}{4} \cdot 8 = \frac{3}{4} \cdot \frac{4}{3}a$ Multiply each side by the reciprocal.

$6 = a$ Simplify.

Check your solution by substituting it in the original equation.

Turn to Page 2 and solve the 18 equations. **Please show your work and circle your answer.**

Solve the equation. (Show your work)

1.) $x + 12 = 25$

2.) $a - 6 = 0$

3.) $-42 = -7c$

4.) $\frac{1}{5}n = 10$

5.) $-32y = 4$

6.) $-\frac{3}{4}x = 24$

7.) $w - 5 = -13$

8.) $4.6 + y = 2.6$

9.) $4a = 132$

10.) $-6 = c + 4$

11.) $-\frac{4}{7}x = -8$

12.) $\frac{2}{3}y = 7$

13.) $\frac{1}{2}x = -40$

14.) $330 = -15c$

15.) $y + 7 = -16$

16.) $\frac{9}{2}x = -1$

17.) $a - \frac{1}{8} = \frac{5}{8}$

18.) $\frac{n}{3} = 6$

SOLVING LINEAR EQUATIONS (MULTI – STEP)

Solving a linear equation may require several steps. You may need to simplify one or both sides of the equation, use the distributive property, or collect variable terms on one side of the equation.

EXAMPLE Solve the equation.

a. $\frac{1}{5}x + 7 = 3$

b. $5 - 2(r + 6) = 1$

SOLUTION

a. $\frac{1}{5}x + 7 = 3$

$\frac{1}{5}x + 7 - 7 = 3 - 7$ **Subtract 7 from each side.**

$\frac{1}{5}x = -4$ **Simplify.**

$5 \cdot \frac{1}{5}x = 5(-4)$ **Multiply by the reciprocal.**

$x = -20$ **Simplify.**

CHECK

$\frac{1}{5}(-20) + 7 = 3$ ✓

b. $5 - 2(r + 6) = 1$

$5 - 2r - 12 = 1$ **Distributive property**

$-2r - 7 = 1$ **Simplify.**

$-2r - 7 + 7 = 1 + 7$ **Add 7 to each side.**

$-2r = 8$ **Simplify.**

$\frac{-2r}{-2} = \frac{8}{-2}$ **Divide each side by −2.**

$r = -4$ **Simplify.**

Solve the equation.

19.) $3y - 4 = 20$

20.) $\frac{c}{7} + 2 = 1$

21.) $6 - \frac{3a}{2} = -6$

22.) $3r - (2r + 1) = 21$

23.) $5(x + 3) = 12$

24.) $44 = 5y - 8 - y$

25.) $75 + 7c = 2c$

26.) $11r + 120 = -r$

27.) $\frac{3}{5}n + 12 = 2n - 9$

28.) $4 - 6p = 2p - 3$

29.) $7(a - 3) = 8a + 2$

30.) $x - (-4x + 2) = 13$

31.) $-\frac{1}{2}(16 - 2y) = 11$

32.) $\frac{1}{4}w + 27 = 41$

33.) $7(4c + 1) - 2(2c - 3) = -23$

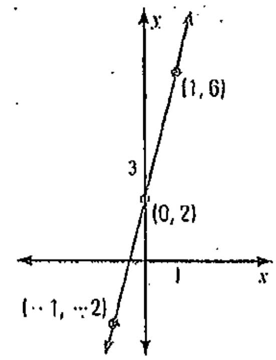
34.) $104 + x = \frac{1}{2}(360 - 2x)$

LINEAR EQUATIONS AND THEIR GRAPHS

A **solution** of an equation in two variables x and y is an ordered pair (x, y) that makes the equation true. Equations like $2x + 3y = -6$, $y = 5x - 1$, and $y = 3$ are **linear equations**. Their graphs are lines.

EXAMPLE Graph the equation $y - 4x = 2$.

SOLUTION You can use a table of values to graph the equation $y - 4x = 2$. Rewrite the equation in *function form* by solving for y: $y - 4x = 2$, so $y = 4x + 2$. Choose a few values of x. Substitute to find the corresponding y-value.

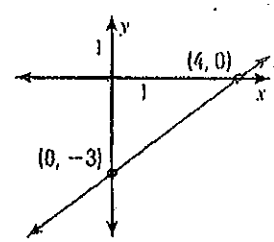


x	$y = 4x + 2$	(x, y)
-1	$y = 4(-1) + 2 = -2$	$(-1, -2)$
0	$y = 4 \cdot 0 + 2 = 2$	$(0, 2)$
1	$y = 4 \cdot 1 + 2 = 6$	$(1, 6)$

Plot the points in the table. Draw a line through the points.

EXAMPLE Graph the equation $3x - 4y = 12$.

SOLUTION You can quickly draw a graph of an equation such as $3x - 4y = 12$ by using the *intercepts*. The **x-intercept** is the x-coordinate of a point where the graph crosses the x-axis. The **y-intercept** is the y-coordinate of a point where the graph crosses the y-axis.



Substitute 0 for x: $3 \cdot 0 - 4y = 12$; $y = -3$, and so the y-intercept is -3.

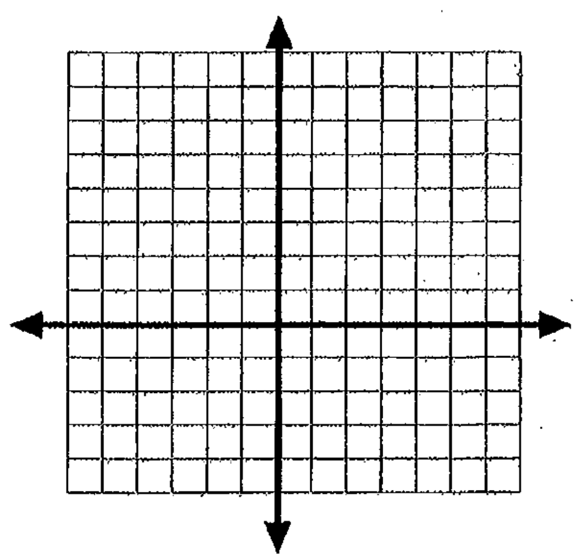
Substitute 0 for y: $3x - 4 \cdot 0 = 12$; $x = 4$, and so the x-intercept is 4.

Now you can graph the equation $3x - 4y = 12$ by plotting the points $(4, 0)$ and $(0, -3)$ and then drawing a line through the points.

Use a table of values to graph the equation. (Show at least four points.)

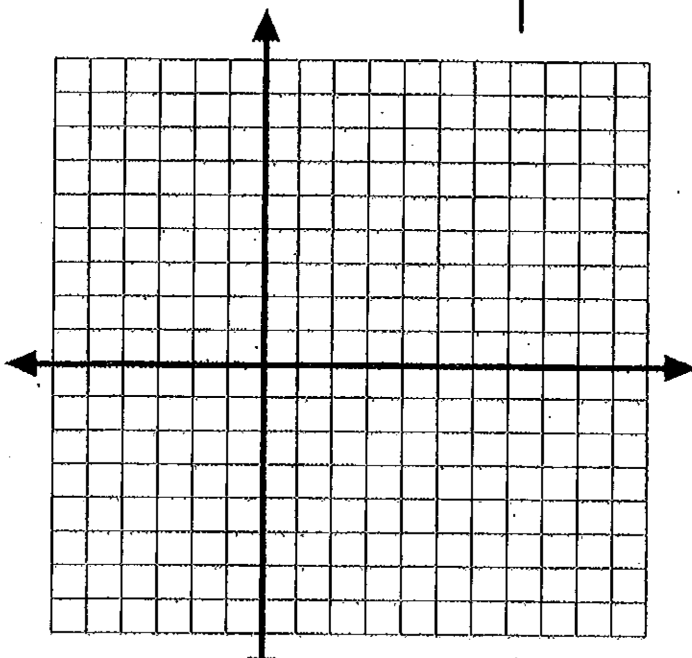
35.) $2x + y = 3$

x	y



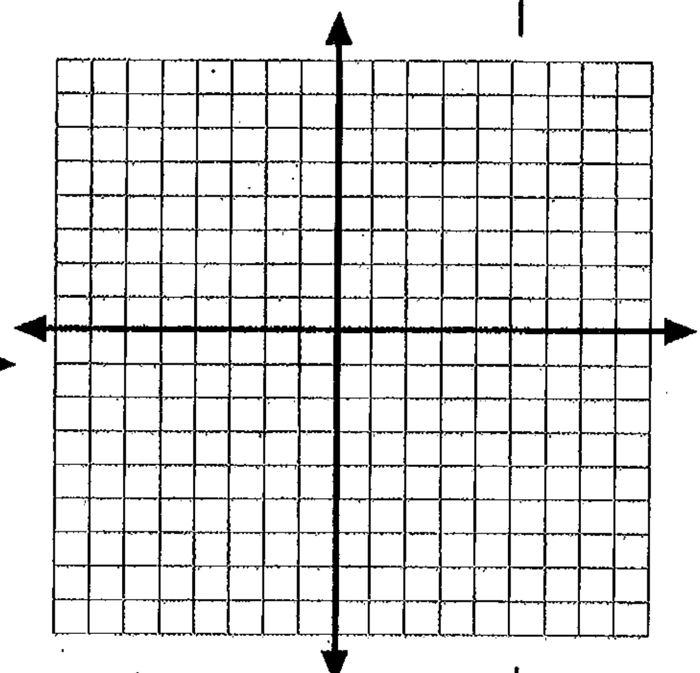
36.) $x - 3y = 6$

x	y



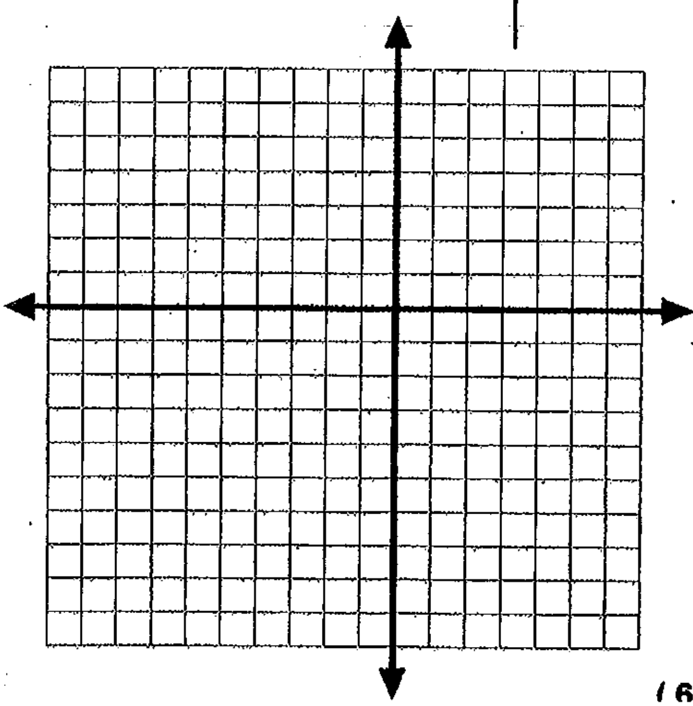
37.) $4x + y = -8$

x	y



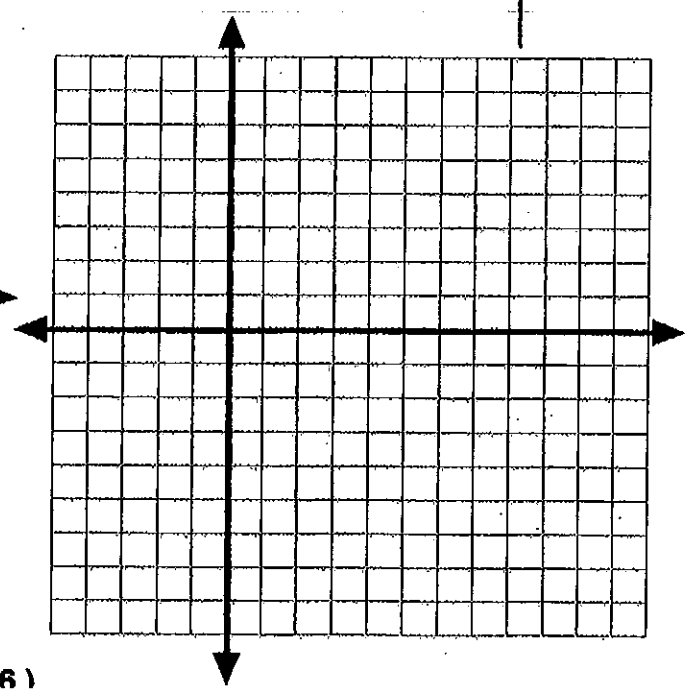
38.) $x + 5y = -10$

x	y



39.) $-\frac{1}{3}x + y = -2$

x	y



SLOPE – INTERCEPT FORM

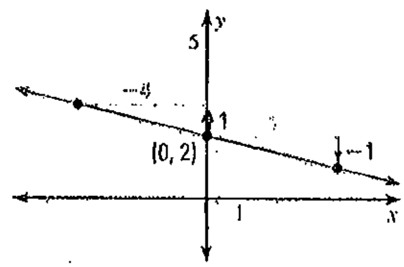
Another way to draw the graph of a linear equation is to use the slope and the y-intercept. Recall that the slope of a nonvertical line is $m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$.

The linear equation $y = mx + b$ is written in **slope-intercept form**. The slope of the line is m and the y-intercept is b.

Graph the equation $\frac{1}{2}x + 2y = 4$.

SOLUTION To graph the equation $\frac{1}{2}x + 2y = 4$, write the equation in slope-intercept form:

$$2y = -\frac{1}{2}x + 4;\ y = -\frac{1}{4}x + 2.$$



The slope m is $-\frac{1}{4}$, and the y-intercept b is 2.

Plot the point (0, 2).

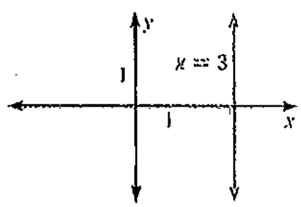
Draw a *slope triangle* to locate a second point on the line:

$$m = -\frac{1}{4} = \frac{\text{rise}}{\text{run}}.$$

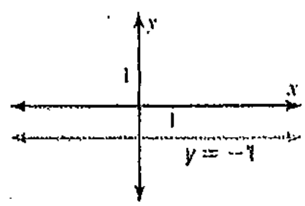
Draw a line through the two points.

Graph $x = 3$ and $y = -1$.

SOLUTION



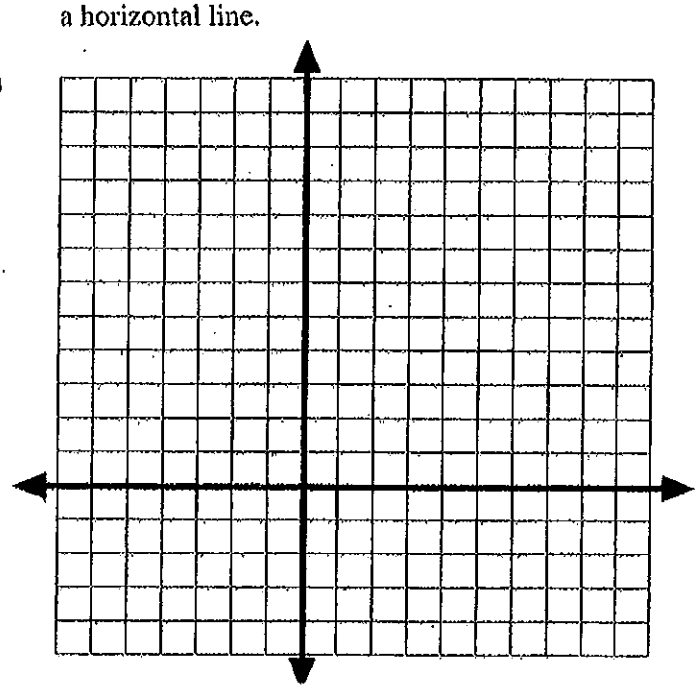
The graph of $x = 3$ is a vertical line.



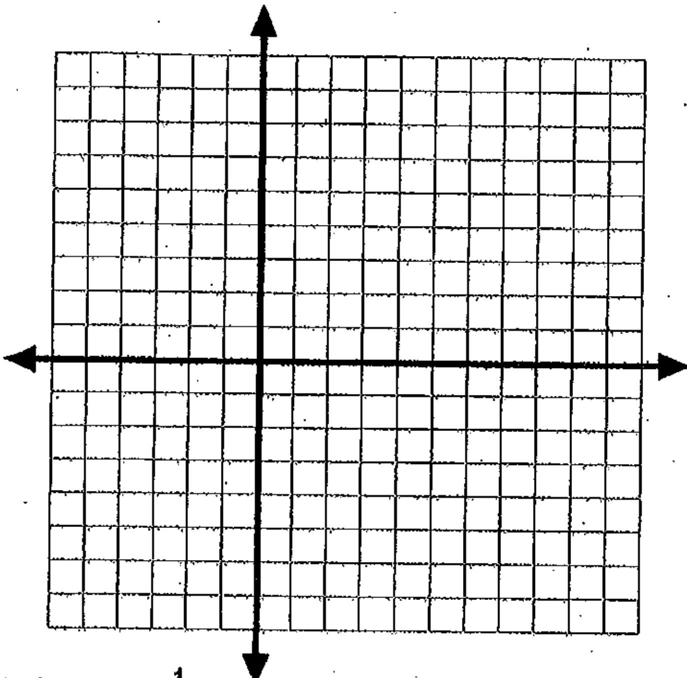
The graph of $y = -1$ is a horizontal line.

Use the slope and the y-intercept to graph the equation.

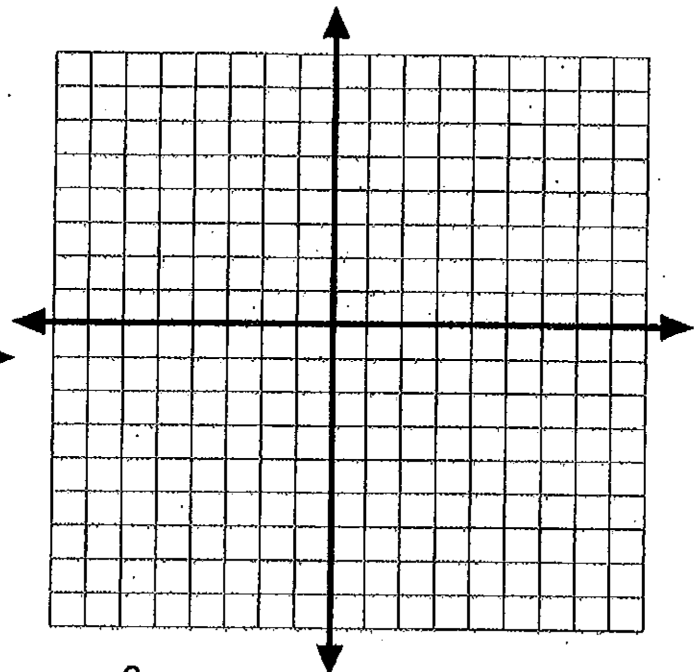
40.) $y = \frac{1}{2}x + 4$



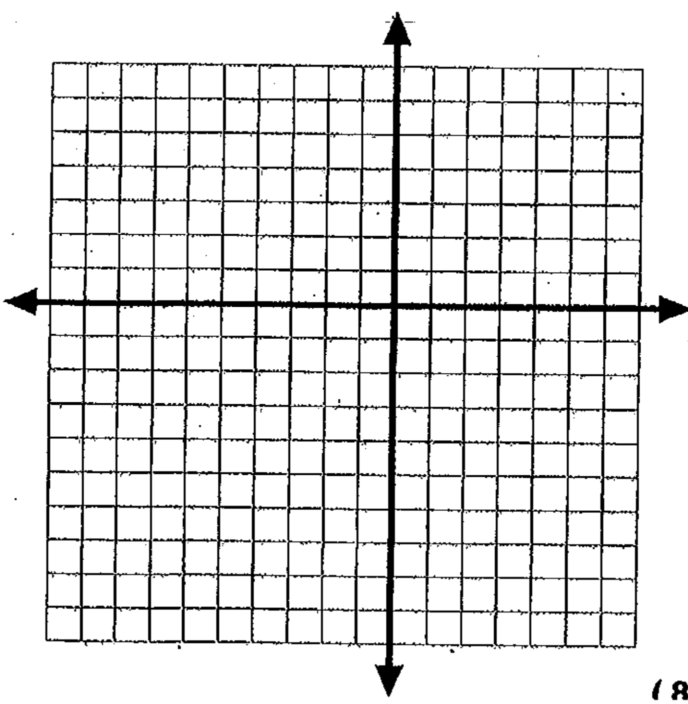
41.) $y = 2x - 4$



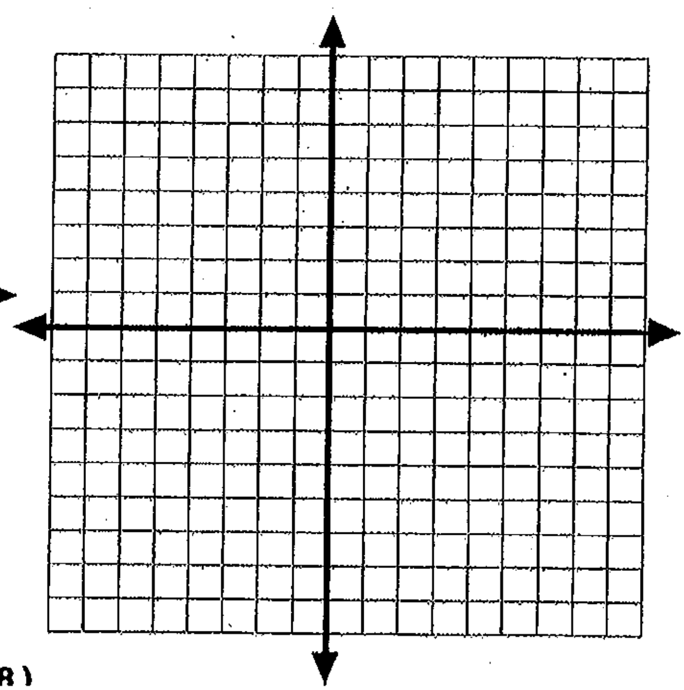
42.) $y = -3x + 5$



43.) $y = \frac{1}{4}x - 7$



44.) $-\frac{2}{3}x + y = 0$

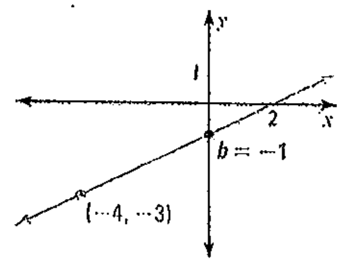


WRITING LINEAR EQUATIONS

The slope of a nonvertical line is $m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$.

Given the slope and the y-intercept of a line, the slope and a point on a line, or two points on a line, you can use the slope-intercept form to write an equation of the line.

EXAMPLE Write an equation of a line that has a slope of $\frac{1}{2}$ and passes through the point $(-4, -3)$.



SOLUTION

$y = mx + b$ — **Write slope formula.**

$-3 = \frac{1}{2}(-4) + b$ — **Substitute.**

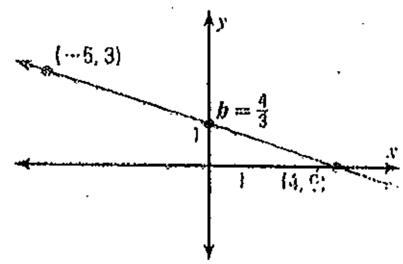
$-1 = b$ — **Simplify.**

▶ So, $m = \frac{1}{2}$ and $b = -1$, and an equation of the line is $y = \frac{1}{2}x - 1$.

EXAMPLE Write an equation of a line that passes through the points $(4, 0)$ and $(-5, 3)$.

SOLUTION First find the slope of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 0}{-5 - 4} = -\frac{1}{3}$$



Then substitute the slope and the coordinates of either point into the slope-intercept formula to find the y-intercept. Let $m = -\frac{1}{3}$, $x = 4$, and $y = 0$.

$0 = -\frac{1}{3} \cdot 4 + b$ — **Substitute.**

$b = \frac{4}{3}$ — **Simplify.**

▶ An equation of the line is $y = -\frac{1}{3}x + \frac{4}{3}$.

Write an equation in slope - intercept form of the line that passes through the given point and has the given slope.

45.) $(0, -4)$, $m = 1$

46.) $(0, 8)$, $m = -3$

47.) $(1, -5)$, $m = 2$

48.) $(3, -11)$, $m = 0$

49.) $(3, 0)$, $m = -4$

50.) $(-6, -6)$, $m = 12$

Write an equation in slope - intercept form of the line that passes through the given 2 points.

51.) (1, 2), (3, – 2)

52.) (0, – 3), (– 5, 0)

53.) (– 6, – 7), (– 5, 1)

54.) (4, 2), (7, – 4)

55.) (11, – 1), (– 1, – 7)

56.) (– 5, 4), (2, – 3)

57.) (4, – 9), (8, – 9)

58.) (– 12, – 13), (3, – 3)

SOLVING SYSTEMS OF EQUATIONS

EXAMPLE Use substitution to solve the linear system: $3x + 2y = 16$ **Equation 1**

$x + 3y = 10$ **Equation 2**

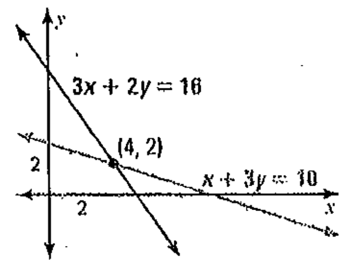
SOLUTION

Solve for x in Equation 2 since it is easy to isolate x: $x = 10 - 3y$.

Substitute $10 - 3y$ for x in Equation 1: $3(10 - 3y) + 2y = 16$.

Solve for y to get $y = 2$. Then $x = 10 - 3y = 10 - 3 \cdot 2 = 4$.

The solution is (4, 2). One way to check this solution is to substitute 4 for x and 2 for y in each of the original equations. Another way is to graph the original equations in the same coordinate plane to see if the graphs intersect at the point (4, 2).



EXAMPLE Use linear combinations to solve the linear system.

$4x - 3y = -5$

$7x + 2y = -16$

SOLUTION The goal is to obtain coefficients that are opposites for one of the variables.

$4x - 3y = -5$ **Multiply by 2.** $8x - 6y = -10$

$7x + 2y = -16$ **Multiply by 3.** $\underline{21x + 6y = -48}$

$29x = -58$ **Add the equations.**

$x = -2$ **Solve for x.**

Substitute -2 for x: $4(-2) - 3y = -5$. Solve to get $y = -1$.

▶ The solution is $(-2, -1)$. Check this in the original equations.

Use substitution to solve the system of linear equations.

59.) $2x - 3y = -16$
$y = 5x + 1$

60.) $3x + 5y = -8$
$4x - y = -3$

61.) $7x - y = 0$
$x - y = 12$

62.) $2x - y = 13$

$y = 3x - 16$

63.) $x + 4y = 30$

$6x + 2y = 4$

64.) $9x + 4y = 3$

$x + 8y = 6$

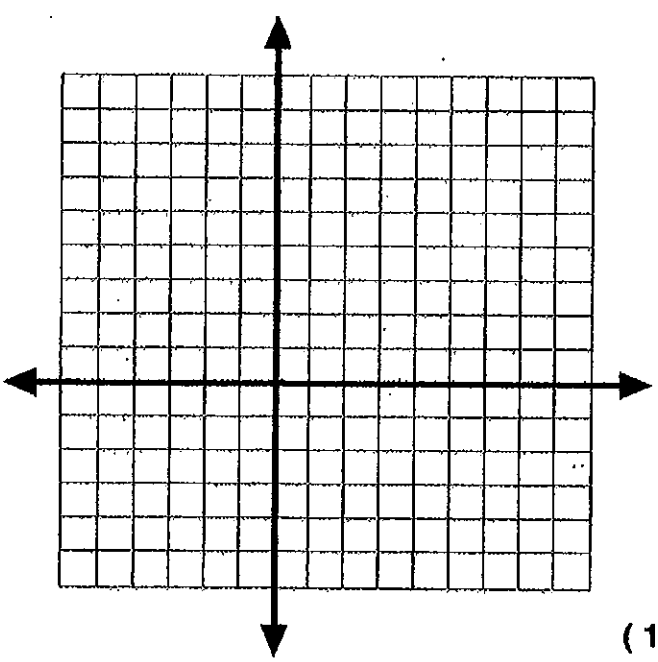
Use linear combinations to solve the system of linear equations. Graph your equations to check your answers.

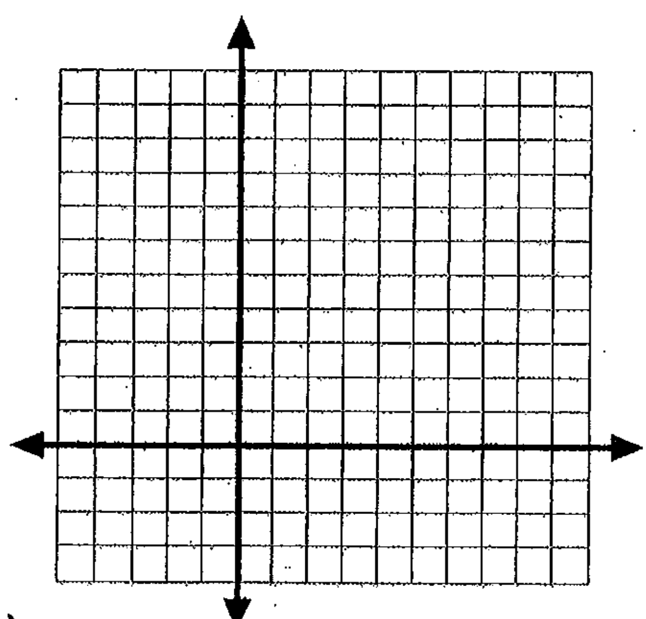
65.) $8x - 2y = 26$

$5x + 2y = 26$

66.) $5x + y = 22$

$4x - 3y = -9$



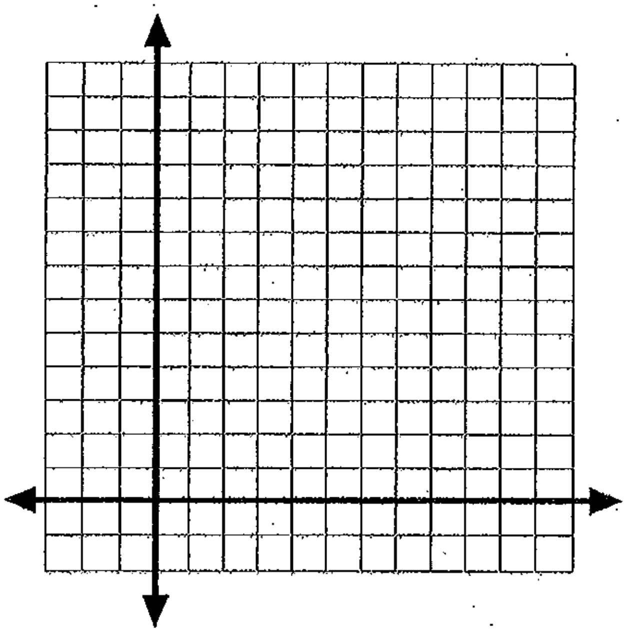


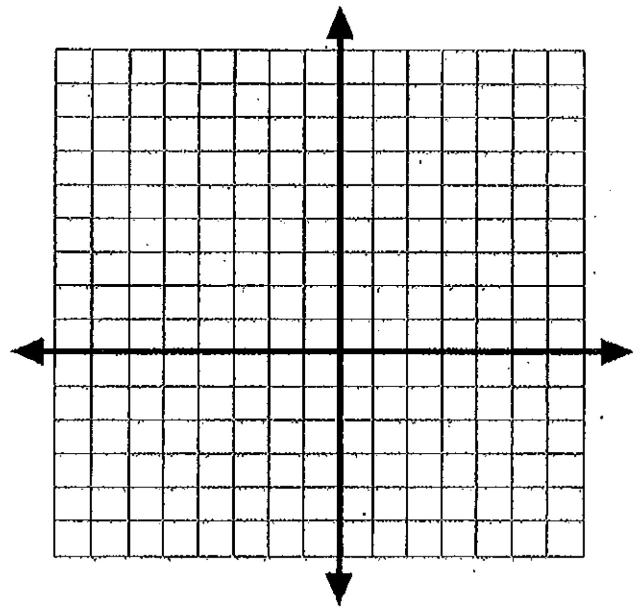
67.) $2x + 3y = 35$

$3x + 4y = 50$

68.) $6x - 2y = -30$

$5x + 4y = -8$

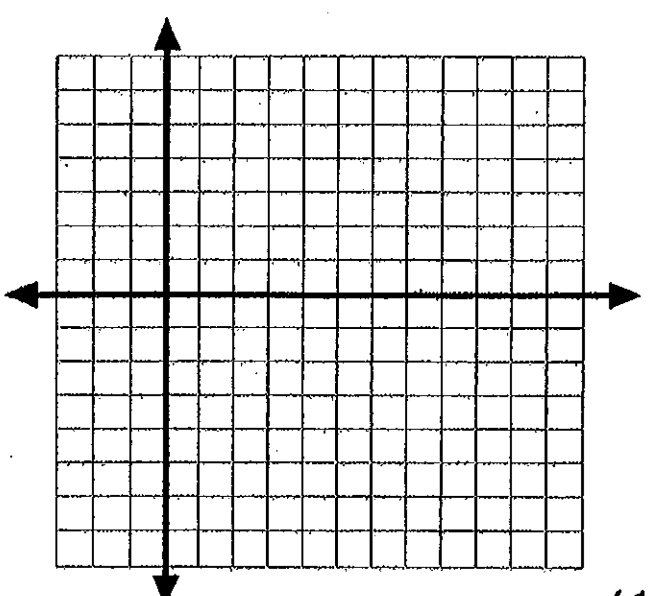




69.) Choose either method to solve the following system. Graph both equations to check your solution.

$3x + 7y = 4$

$y = -\frac{1}{3}x$



SIMPLIFYING RADICALS

EXAMPLE 1

Simplify the expression $\sqrt{20}$.

$\sqrt{20} = \sqrt{4} \cdot \sqrt{5}$ **Use product property.**

$= 2\sqrt{5}$ **Simplify.**

EXAMPLE 2

Simplify the radical expression.

a. $(5\sqrt{7})^2$

$= 5^2\sqrt{7^2}$

$= 25 \cdot 7$

$= 175$

b. $(2\sqrt{2})(5\sqrt{3})$

$= 2 \cdot 5 \cdot \sqrt{2} \cdot \sqrt{3}$

$= 10\sqrt{6}$

Simplify the expresssion.

70.) $\sqrt{16}$

71.) $\sqrt{49}$

72.) $\sqrt{100}$

73.) $\sqrt{12}$

74.) $\sqrt{40}$

75.) $\sqrt{27}$

76.) $\sqrt{80}$

77.) $\sqrt{72}$

78.) $(3\sqrt{5})^2$

79.) $5\sqrt{45}$

80.) $2\sqrt{8}$

81.) $(3\sqrt{3})(2\sqrt{6})$

82.) $(4\sqrt{3})^2$

83.) $6\sqrt{90}$

84.) $(4\sqrt{6})(2\sqrt{10})$

SOLVING QUADRATIC EQUATIONS

A **quadratic equation** is an equation that can be written in the **standard form** $ax^2 + bx + c = 0$ where $a \neq 0$.

When $b = 0$, the quadratic equation has the form $ax^2 + c = 0$. In this case, you can solve for x. Solving $ax^2 + c = 0$ for x^2 you get $x^2 = \frac{-c}{a}$ and the following rules apply.

- If $\frac{-c}{a} > 0$, then $x^2 = \frac{-c}{a}$ has two solutions, $x = \sqrt{\frac{-c}{a}}$ and $x = -\sqrt{\frac{-c}{a}}$.
- If $\frac{-c}{a} = 0$, then $x^2 = \frac{-c}{a}$ has one solution, $x = 0$.
- If $\frac{-c}{a} < 0$, $x^2 = \frac{-c}{a}$ has no real solution.

EXAMPLES Solve the equation.

a. $3x^2 - 1 = 23$ **b.** $12 - x^2 = 13$ **c.** $4 + 2n^2 = 4$

SOLUTION

a. $3x^2 - 1 = 23$

$3x^2 = 24$

$x^2 = 8$

$x = \pm\sqrt{8}$

$x = \pm 2\sqrt{2}$

b. $12 - x^2 = 13$

$-x^2 = 1$

$x^2 = -1$

no real solution

c. $4 + 2n^2 = 4$

$2n^2 = 0$

$n^2 = 0$

$n = 0$

EXAMPLE Solve $(x + 2)^2 = x^2 + 9$.

SOLUTION

$(x + 2)^2 = x^2 + 9$

$x^2 + 4x + 4 = x^2 + 9$

$4x = 5$

$x = 1.25$

Solve the equation. Round your solutions to the nearest hundredth.

85.,) $x^2 = 100$

86.) $4x^2 = 80$

87.) $3x^2 - 8 = 4$

88.) $x^2 + 6 = 11$

89.) $\frac{1}{2}x^2 + 3 = 245$

90.) $(x + 1)^2 = 27 + x^2$

You can solve any quadratic equation by using the **quadratic formula.** This formula, which you used in Algebra, states that the solutions of the quadratic equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ when } a \neq 0 \text{ and } b^2 - 4ac \geq 0.$$

EXAMPLE Solve $x^2 - 4x - 12 = 0$ by using the quadratic formula.

SOLUTION Substitute $a = 1$, $b = -4$, and $c = -12$ in the quadradic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-12)}}{2 \cdot 1} = \frac{4 \pm \sqrt{64}}{2}$$

▶ The solutions are $\frac{4+8}{2} = 6$ and $\frac{4-8}{2} = -2$.

Check your solutions by substituting each solution into the original equation.

$6^2 - 4(6) - 12 = 0$ $\quad$ $(-2)^2 - 4(-2) - 12 = 0$

$36 - 24 - 12 = 0$ ✓ $\quad$ $4 + 8 - 12 = 0$ ✓

EXAMPLE Solve $2x^2 + 6x = 1$ by using the quadratic formula.

SOLUTION

Begin by writing the equation in *standard form*: $2x^2 + 6x - 1 = 0$.

Substitute $a = 2$, $b = 6$, and $c = -1$ in the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{6^2 - 4 \cdot 2(-1)}}{2 \cdot 2} = \frac{-6 \pm \sqrt{44}}{4} = \frac{-6 \pm 2\sqrt{11}}{4} = \frac{-3 \pm \sqrt{11}}{2}$$

▶ The solutions are $\frac{-3 + \sqrt{11}}{2} \approx 0.16$ and $\frac{-3 - \sqrt{11}}{2} \approx -3.16$.

Use the quadratic formula to solve each equation. Round your solutions to the nearest hundredth.

91.) $x^2 + 5x + 4 = 0$

92.) $2x^2 + 4x + 1 = 0$

93.) $3x^2 + 6x = -2$

94.) $x^2 + 10x = -25$

EXAMPLE

THROWN BALL You throw a ball up into the air. At 4 feet above the ground, the ball leaves your hand with an initial vertical velocity of 30 feet per second.

a. Write an equation that gives the height (in feet) of the ball as a function of the time (in seconds) since it left your hand.

b. After how many seconds does the ball land on the ground?

Solution

a. Use the vertical motion model $h = -16t^2 + vt + s$ to write an equation for the height h (in feet) of the ball as a function of the time t (in seconds). In this case, $v = 30$ and $s = 4$.

$h = -16t^2 + vt + s$ **Vertical motion model**

$h = -16t^2 + 30t + 4$ **Substitute 30 for v and 4 for s.**

b. When the ball lands on the ground, its height is 0 feet. Substitute 0 for h and solve the equation for t.

$0 = -16t^2 + 30t + 4$ **Substitute 0 for h.**

$0 = -2(8t^2 - 15t - 2)$ **Factor out −2.**

$0 = -2(8t + 1)(t - 2)$ **Factor the trinomial. Find factors of 8 and −2 that produce a middle term with a coefficient of −15.**

$8t + 1 = 0$ *or* $t - 2 = 0$ **Zero-product property**

$t = -\frac{1}{8}$ *or* $t = 2$ **Solve for t.**

The solutions of the equation are $-\frac{1}{8}$ and 2. A negative solution does not make sense in this situation, so disregard $-\frac{1}{8}$.

▶ The ball lands on the ground after 2 seconds.

9.5 Factor $x^2 + bx + c$

pp. 583–589

EXAMPLE

Factor $x^2 + 2x - 63$.

Find two factors of −63 whose sum is 2. One factor will be positive, and the other will be negative. Make an organized list of factors.

Factors of −63	Sum of factors	
1, −63	1 + (−63) = −62	✗
−1, 63	−1 + 63 = 62	✗
3, −21	3 + (−21) = −18	✗
−3, 21	−3 + 21 = 18	✗
9, −7	9 + (−7) = 2	← Correct sum
−9, 7	−9 + 7 = −2	✗

▶ $x^2 + 2x - 63 = (x + 9)(x - 7)$

Make sure you check to see if ***a=1***. Factor each expression completely.

1. $x^2 - x - 6$

2. $4x^2 - 4x + 1$

3. $4x^2 + 14x - 8$

4. $4x^2 - 16$

5. $x^2 - 36$

6. $x^2 + 15x + 54$

7. $x^2 + 8x + 16$

8. $12x^2 + 14x - 6$

9. $9x^2 - 4$

10. $x^2 - 9$

11. $15x^2 - 5x - 10$

12. $x^2 - 6x +$